

# The Inapplicability of Evolutionarily Stable Strategy to the Prisoner's Dilemma

LOUIS MARINOFF

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## ABSTRACT

The Axelrod-Hamilton games-theoretic conflict model, which applies Maynard Smith's concept of evolutionarily stable strategy to the Prisoner's Dilemma, gives rise to an inconsistency between theoretical prescription and empirical results. Proposed resolutions of this problem are incongruent with the tenets of the models involved. The independent consistency of each model is restored, and the anomaly thereby circumvented, by a proof that no evolutionarily stable strategy exists in the Prisoner's Dilemma.

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## I INTRODUCTION

The formal theory of games is a genuinely new branch of mathematics, created and construed by Von Neumann and Morgenstern as a calculus of rational decision-making, or behaviour, in situations of risk or conflict of interest. Though the title of their work suggests that they intended the theory to be applied primarily to economics (see Von Neumann and Morgenstern [1944]), it has since been widely extended to diverse areas such as social psychology, political science, decision theory, conflict resolution, moral philosophy, and evolutionary theory. While these and other applications amply testify to the richness of the theory, problems can arise when extensions of it, made in critically different fields, are methodologically synthesized or otherwise re-united.

We shall examine one such exercise—namely, Axelrod and Hamilton's [1981] incorporation of evolutionarily stable strategy into the Prisoner's Dilemma. A re-examination of the mathematical foundations of evolutionary

games will reveal that their rationale for co-mingling these two branches of games theory reposes upon a faulty assumption.

We begin by tracing the two separate branches of Axelrod and Hamilton's eventual synthesis.

## 2 THE PRISONER'S DILEMMA

First, consider the standard Prisoner's Dilemma (*e.g.* Axelrod [1981]):

	P <sub>2</sub>	
	<i>c</i>	<i>d</i>
C	R,R	S,T
P <sub>1</sub>		
D	T,S	P,P

where  $T > R > P > S$

and  $R > (1/2)(S + T)$

for P<sub>1</sub> (player one): C means co-operate, D means defect

for P<sub>2</sub> (player two): *c* means co-operate, *d* means defect

The dilemma results from divergent dictates of two principles of choice: dominance versus maximization of expected utility. Applying the dominance principle, P<sub>1</sub> reasons that, no matter what P<sub>2</sub> does, P<sub>1</sub> is better off defecting since, in terms of possible payoffs,  $T > R$  and  $P > S$ . Thus defection is strongly dominant.<sup>1</sup> The argument is symmetric for P<sub>2</sub>. Hence both players defect, to their mutual detriment.

On the other hand, a player may compute the expected utility (*EU*) of each row (or column) of the game matrix, and may select the row (or column) for which the *EU* is a maximum. For a given row (or column), the *EU* is the sum of the products of the utility (*U*) of each game-state and the probability of that game-state occurring. Most generally, if a given row (or column) has *n* states, and if any state *v<sub>i</sub>* obtains with probability *p<sub>i</sub>*, then the expected utility of that row (or column) is given by

$$EU = \sum_{i=1}^n p_i U(v_i) \quad (1a)$$

In our case, P<sub>1</sub>'s expected utilities are:

$$EU(C) = p(c/C)R + p(d/C)S \quad (1b)$$

$$EU(D) = p(c/D)T + p(d/D)P \quad (1c)$$

For  $p(c/C)$  read: 'the probability that P<sub>2</sub> co-operates conditional on the assumption that P<sub>1</sub> co-operates'. (This notation is from Campbell and Sowden

<sup>1</sup> Action A strongly dominates action B if and only if, for each game-state, P prefers the consequences of A to those of B. (*E.g.* see Nozick [1985].)

[1985], p. 18.) If we assume complete probabilistic dependence,<sup>2</sup>  $p(c/C) = p(d/D) = 1$ , and  $p(d/C) = p(c/D) = 0$ . Then  $EU(C) = R$  and  $EU(D) = P$ . Since  $R > P$ , maximization of expected utility prescribes co-operation. The argument is symmetric for  $P_2$ . Hence both players co-operate, to their mutual benefit.

What concerns us here are the results of two tournaments conducted by Axelrod [1980a, b], in which the standard Prisoner's Dilemma was played by computer programs submitted, in the main, by academicians whose backgrounds include expertise in games theory. Each program was run against its twin, round-robin against the others, and against a program that played randomly. Anatol Rapoport's submission, called 'Tit-for-Tat' (*TFT*) won both tournaments. *TFT* is the games-theoretic equivalent of *lex talionis*: it co-operates on the first move, and plays next whatever its opponent played last. It therefore swiftly punishes defection, and equally swiftly rewards co-operation. *TFT* is the best strategy yet devised for iterated Prisoner's Dilemmas. But we draw urgent attention to Axelrod's [1980b] observation: 'There is no best strategy independent of environment.'

### 3 THE EVOLUTIONARILY STABLE STRATEGY

The second model under consideration owes to Maynard Smith's fruitful application of games theory to evolutionary theory. While Lewontin [1961] first conceived of a 'game against nature' (in applying the minimax criterion to population genetics), Maynard Smith [1974] modelled phenotypic conflict, in the animal kingdom exclusive of humans, in games-theoretic terms. Essentially, he proposed that if two members of a species compete for a fitness-enhancing resource of expected utility  $V$ , they may adopt either the 'hawk' strategy ( $H$ ) and attempt to monopolize it, or the 'dove' strategy ( $D$ ) and share it. The payoff matrix is as follows (see Maynard Smith [1982], p. 12):

		$P_2$	
		$H$	$D$
	$H$	$(1/2)(V - C)$	$V, 0$
$P_1$			
	$D$	$0, V$	$(1/2)V$

where  $C$  denotes loss of fitness due to injurious effect of  $\{H, H\}$  conflict.

(Note:  $C$  and  $D$ , in this model, are not to be confused with their signification

<sup>2</sup> This assumption is strongly supported in the literature of the Prisoner's Dilemma. An inductive argument is given by Rapoport [1966], pp. 145-57. Empirically, the tendency toward joint similar play, i.e. either  $\{C, c\}$  or  $\{D, d\}$ , increases with the length of the game (see Rapoport [1969], p. 157).

in the Prisoner's Dilemma. We have retained the familiar notations for both models, and symbols common to both should be interpreted in their respective contexts.)

It is immediately apparent that this model embraces a complexity absent from the Prisoner's Dilemma, in that the utility of the  $\{H, H\}$  game-state may be either positive (if  $V > C$ ), zero (if  $V = C$ ), or negative (if  $V < C$ ). The relative merits of possible strategies will differ according to which case obtains.

In the event that  $V > C$ , the situation resembles a Prisoner's Dilemma. The 'hawk' strategy is strongly dominant, since  $(1/2)(V - C) > 0$  and  $V > (1/2)V$ , while, again assuming complete probabilistic dependence, maximization of expected utility prescribes the 'dove' strategy, since  $EU(D) = (1/2)V > EU(H) = (1/2)(V - C)$ . Similarly, if  $V = C$ , the 'hawk' strategy is weakly dominant,<sup>3</sup> while maximization of expected utility continues to prescribe the 'dove' strategy. Be that as it may, animals (exclusive of humans) do not appear to avail themselves of a calculus of expected utilities, and in cases where  $V \geq C$ , the 'hawk' strategy prevails. This, in effect, is also asserted by Maynard Smith ([1982], p. 15).

In the third case, for which  $V < C$ , the evolutionary model exhibits novel properties. Predacious animals, being armed with a variety of weapons lethal to con-specifics as well as to their prey, are in a predicament in which con-specific conflict can be debilitating or fatal to one or both of the competitors. In order to avoid such drastic reductions in individual fitness, which might also prove inimical to the survival of a given species, natural selection (it is presumed) has favoured the evolution of ritualized or limited combat. Thus, for example, certain venomous serpents wrestle against one another instead of using their deadly fangs (see Maynard Smith and Price [1973]). Clearly, when  $V < C$ , no 'pure' strategy (consisting exclusively of either 'hawk' or 'dove' behaviour) is viable. How is such behaviour to be reflected in the game-theoretic model?

In many human games, 'mixed' strategies are optimally effective—as, for instance, is *TFT* in the Prisoner's Dilemma. Moreover, there exists a very large class of games in which the optimal strategy is not only mixed, but also in which the mixture itself is achieved by random (or pseudo-random) methods, so that an opponent can neither detect nor exploit any strategically predictable pattern of play.<sup>4</sup> Animal behaviour is highly purposive. It is shaped by a welter of physical, chemical and biological gradients, and is tempered by experience. But animals (exclusive of humans) do not make use of decision theory; and

<sup>3</sup> Action A weakly dominates action B if and only if, for each game-state, P either prefers the consequences of A to those of B or is indifferent between the consequences; and for some game-state, P prefers the consequence of A to that of B. (E.g. see Nozick [1985].)

<sup>4</sup> Technically, a mixed strategy of random play is prescribed in games that are not strictly determined; that is, in any game whose matrix does not contain a saddle-point. This follows from the fundamental theorem of the functional calculus of two-person, zero-sum games. (See Von Neumann and Morgenstern [1944], pp. 95–146.)

when their strategies are mixed, the distribution within the mixture cannot be described as random.

So, for the prodigious number of species to which the case  $V < C$  applies, Maynard Smith ([1982], p. 15) advanced the following hypothesis: suppose that there exists some strategy  $I$ , which consists in playing  $H$  with probability  $p$ , and in playing  $D$  with probability  $(1-p)$ , and suppose that said probability is determined by a genome which carries the relative frequency of phenotypic behaviours  $H$  and  $D$ . In terms of the evolutionary stability of a given species, an optimal behavioural phenotype is positively selected, and the genotype that mediates this strategic distribution is thereby preserved and transmitted. This optimal mixed strategy  $I$ , for a given species, is called an evolutionarily stable strategy (ESS). An ESS is defined (by Maynard Smith and Price [1973]) as a strategy such that: 'if most of the members of a population adopt it, there is no "mutant" strategy that would give higher reproductive fitness'. An important alternate definition is given by Maynard Smith ([1982], p. 10): 'An ESS is a strategy such that, if all the members of a population adopt it, then no mutant strategy could invade the population under the influence of natural selection.'

Now, to find  $p$  such that  $I$  is an ESS, Maynard Smith ([1982], pp. 15-16) employed the useful result of the Bishop-Cannings [1978] theorem,<sup>5</sup> and solved the following equation:

$$EU(H,I) = EU(D,I) \quad (2a)$$

(This might be termed the 'Shroedinger equation' of evolutionary games theory.) Explicitly:

$$p[(1/2)(V-C)] + (1-p)V = p(0) + (1-p)(1/2)V \quad (2b)$$

or

$$p = V/C \quad (2c)$$

Thus, when  $V < C$ , it is evolutionarily stable to adopt the 'hawk' strategy with probability  $V/C$ , and the 'dove' strategy with probability  $1 - (V/C)$ . It is also possible to sustain a stable, genetically polymorphic population of pure-breeding 'hawks' and 'doves', but their respective stable ratio will be identically  $V/C$  (see Maynard Smith [1982], pp. 16-17).

Having sketched the strategic possibilities arising from the three cases of this evolutionary games model, we note a significant caveat cited by Maynard Smith [1974]; namely, that although natural selection is presumed to act upon the individual animal, 'the success of any particular strategy depends upon what strategies are adopted by other members of the population.' Now recall Axelrod's [1980b] statement concerning iterated Prisoner's Dilemmas:

<sup>5</sup> Bishop and Cannings [1978] showed that if  $I$  is a mixed ESS with component strategies  $a, b, c, \dots$  (called the 'support of  $I$ '), then  $EU(a,I) = EU(b,I) = EU(c,I) \dots = EU(I,I)$ .

'There is no best strategy independent of environment.' While these two remarks are strongly analogous, a strong analogy is a frail bridge of reason. Although this analogy seems to link the two models, it is a bridge that will not bear the weight of the subsequent analysis.

#### 4 THE AXELROD-HAMILTON MODEL

Our synopses of these two models, however brief, have reviewed sufficient fundamentals for the next task, which is to identify and elucidate the flaw in their entanglement. Axelrod and Hamilton [1981] imported the concept of *ESS* into the Prisoner's Dilemma. Maynard Smith ([1982], pp. 202-3) has endorsed and reproduced their results. In essence, their discussion devolves about finding conditions whereby certain strategies, notably *TFT* and the strategy of pure defection (*DDD*), may be deemed 'evolutionarily stable'.

To achieve this purpose, one must initially establish the expected utilities of repeated encounters between various strategies in the Prisoner's Dilemma. Axelrod and Hamilton [1981] accomplish this by the artful expedient of assuming a constant probability of interaction,  $w$ , among them. The first encounter is 'on the house'; the probability of a second encounter is  $w$ ; of a third,  $w^2$ ; and of an  $n^{\text{th}}$  encounter,  $w^{n-1}$ . Thus the expected utility of *TFT* versus *TFT* (to either player) is

$$EU(TFT, TFT) = R + wR + w^2R + \dots + w^nR = R/(1-w) \quad (3)$$

which is the sum of the geometric series as  $n$  increases without bound.

Similarly, one finds the expected utility of *DDD* versus *DDD*:

$$EU(DDD, DDD) = P + wP + w^2P + \dots + w^nP = P/(1-w); \quad (4)$$

of *DDD* versus *TFT*:

$$EU(DDD, TFT) = T + wP + w^2P + \dots + w^nP = T + wP/(1-w); \quad (5)$$

and of *TFT* versus *DDD*:

$$EU(TFT, DDD) = S + wP + w^2P + \dots + w^nP = S + wP/(1-w). \quad (6)$$

Now, Maynard Smith ([1982], pp. 13-14) has furnished two equations (in his evolutionary games model) with which one can test whether a popular strategy,  $I$ , can be invaded by a mutant strategy,  $J$ . If all individuals have initial fitness  $W_0$  before a contest, and if  $W(I)$  and  $W(J)$  are the resultant fitnesses of adopting popular and mutant strategies, respectively, and if  $p$  is the proportion of  $I$ -strategists in the population, then

$$W(I) = W_0 + (1-p)EU(I, I) + pEU(I, J) \quad (7a)$$

and

$$W(J) = W_0 + (1 - p)EU(J,I) + pEU(J,J) \quad (7b)$$

where  $p \ll 1$ .

If  $I$  is an ESS then, by definition,  $W(I) > W(J)$ . Then it must be the case that either

$$EU(I,I) > EU(J,I) \quad (8a)$$

or

$$EU(I,I) = EU(J,I) \quad \text{and} \quad EU(I,J) > EU(J,J) \quad (8b)$$

From these conditions, it readily follows that the 'dove' strategy is not an ESS, since  $EU(D,D) < EU(H,D)$ . Thus Maynard Smith ([1982], p. 14) observes that 'a population of Doves can be invaded by a Hawk mutant'. By the same token, the 'hawk' strategy is an ESS, provided of course that  $V > C$ .

After having found the expected utilities of combinations of the competing strategies *TFT* and *DDD*, Axelrod and Hamilton [1981] apply them to equations (8a) and (8b) in order to determine the conditions for an ESS in the Prisoner's Dilemma.

First, can *DDD* be invaded by *TFT*? If *DDD* is an ESS and *TFT* a mutant strategy, then, by equations (8a) and (8b), either

$$EU(DDD,DDD) > EU(TFT,DDD) \quad (9a)$$

or

$$EU(DDD,DDD) = EU(TFT,DDD) \quad \text{and} \quad EU(DDD,TFT) > EU(TFT,TFT) \quad (9b)$$

From equations (4) and (5), the inequality (9a) becomes:

$$P/(1 - w) > S + wP/(1 - w) \quad (10a)$$

which reduces to

$$1 > w \quad (10b)$$

Of course, if  $w = 1$ , i.e. if the probability of repeated encounters is unity, equations (3) through (6) do not have finite sums. But for all other  $w$  such that  $0 < w < 1$ , the inequality (9a) is satisfied, and thus *DDD* cannot be invaded by *TFT*. So Axelrod and Hamilton [1981] conclude: 'For any value of  $w$ , [except, we add, for  $w = 1$ ] the strategy of unconditional defection . . . is evolutionarily stable.'

Second, can *TFT* be invaded by *DDD*? If *TFT* is an ESS and *DDD* a mutant strategy, then, again by equations (8a) and (8b), either

$$EU(TFT,TFT) > EU(DDD,TFT) \quad (11a)$$

or

$$EU(TFT,TFT) = EU(DDD,TFT) \quad \text{and} \quad EU(TFT,DDD) > EU(DDD,DDD) \quad (11b)$$

From equations (3) and (5), the inequality (11a) becomes:

$$R/(1-w) > T + wP(1-w) \quad (12a)$$

which reduces to

$$w > (T-R)/(T-P) \quad (12b)$$

But inequality (12b) is not necessarily true for all  $w$ , and is satisfied only if  $w$  is sufficiently large (relative to some fixed values of  $T, R, P, S$ ). Furthermore, the conjunction (11b) cannot be satisfied, since  $EU(TFT, TFT) \neq EU(DDD, TFT)$ . Thus *TFT* can be invaded by *DDD*. So Axelrod and Hamilton [1981] conclude that *TFT* is an *ESS* 'if and only if the interactions between the individuals have a sufficiently large probability of continuing'.

## 5 AN UNSOLVED PROBLEM

Consequent upon these findings, the incorporation of *ESS* into the Prisoner's Dilemma raises a fundamental problem accompanied by several corollary difficulties.

The problem is the discrepancy between the theoretical assertion that *DDD* is an *ESS* in the Prisoner's Dilemma, and the empirical result of Axelrod's tournaments, in which *TFT* proved its relative superiority. If, with some hand-waving, one attempts to account for the anomaly by re-introducing the maxim that the 'best' strategy is environment-dependent, one merely begs the question: why do the theoretical and the empirical environments support different optimal strategies?

Second, one may venture the explanation that, if  $w$  is sufficiently large, *DDD* cannot invade *TFT*, and thus *TFT* is an *ESS*. Unfortunately, this explanation is unsound when applied to the case where it is most needed. For in Axelrod's tournaments,  $w$  is more than 'sufficiently large', it is too large: the probability of repeated encounters is unity for a prescribed number of encounters (and zero thereafter). Since unity is the value of  $w$  for which the infinite series summations of expected utilities are undefined, the theory cannot explain the experimental result in this case. The first problem remains.

Third, Axelrod and Hamilton are well aware of this problem. Their proposed solution consists in representing the emergence of the *ESS* of *TFT* in the Prisoner's Dilemma as a special instance of a more general problem in evolutionary theory, namely altruism, which is universally 'solved' by an appeal to kinship. That the consistency of their model reverts to dependence upon the solution of the central problem of sociobiology (see Wilson [1975], p. 3.), is neither surprising nor convincing. It is not surprising since one of the authors (Hamilton [1964a, b]) single-handedly rescued sociobiology from the paradigm-threatening prospect of Hymenopteran altruism by introducing the very concept of kinship, or genetic relatedness. It is not convincing because, by



definition, an evolutionarily stable strategy cannot be invaded under the influence of natural selection. Thus, if *DDD* is an ESS, *TFT* cannot invade it by kinship. Axelrod and Hamilton [1981], having immersed their problem in that primordial sea of struggle which so inspired Hobbes, Malthus and Darwin, justify it with the postulate that pure defection was the 'primeval state' of nature, and 'solve' it by the contradictory assertion that *TFT* emerged either by kinship or by sufficiently large *w*; in other words, was countenanced by natural selection, which is impossible by definition.

## 6 A SOLUTION

We shall attempt to resolve the main problem, and its corollary contradictions, in one stroke. The calculus of evolutionary games theory, so neatly tricked out in the Prisoner's Dilemma, rests tacitly on the assumptions that *DDD* and *TFT* can be treated as evolutionarily stable strategies. By testing these assumptions via the Maynard Smith equation (2a), it can readily be shown that there is no possible ESS in the Prisoner's Dilemma. The proof is as follows:

Recall the Maynard Smith equation (2a), for the Hawk-Dove game. If *I* is an ESS, then

$$EU(H,I) = EU(DI)$$

Similarly, if *I* is an ESS in the Prisoner's Dilemma, it must be the case that

$$EU(D,I) = EU(C,I) \quad (13a)$$

(Note: in equation (13a) the notation refers to the Prisoner's Dilemma; i.e., *D* means defect and *C* means co-operate.)

Explicitly, as in equation (2b), to find probability *x* such that *I* is an ESS, (13a) becomes:

$$xP + (1-x)T = xS + (1-x)R \quad (13b)$$

Recall that  $T > R > P > S$ , and that  $R > (1/2)(S+T)$ .

Then let  $R = (1/2)(S+T) + (1/2)Z$ , where  $Z > 0$ .

For convenience (and with no significant loss of generality), let  $P = (1/2)(S+T)$ . Then

$$x = (R - T) / [(R + P) - (S + T)] = (R - T) / (R - P)$$

or

$$x = 2(R - T) / Z \quad (13c)$$

thus

$$x < 0 \quad (13d)$$

We cannot, of course, admit a negative probability.

But there exists an alternate formulation of equations (2a) and (13a). In the Hawk-Dove game, if we interchange the rows and columns, we obtain a new matrix, equivalent to rotating the original matrix through 180°:

$$\begin{array}{cc}
 & P_2 \\
 & \begin{array}{cc} D & H \end{array} \\
 P_1 \begin{array}{c} D \\ H \end{array} & \begin{array}{cc} (1/2)V & 0, V \\ V, 0 & (1/2)(V-C) \end{array}
 \end{array}$$

Now equation (2a) is rewritten:

$$EU(D, I) = EU(H, I) \quad (2a')$$

Explicitly, to find  $p$  such that  $I$  is an ESS, we must now solve

$$p(1/2)V + (1-p)(0) = pV + (1-p)[(1/2)(V-C)] \quad (2b')$$

or

$$p = 1 - (V/C) \quad (2c')$$

This is the expected result. The ESS still prescribes playing 'dove' with probability  $1 - (V/C)$ , therefore playing 'hawk' with probability  $V/C$ . The Maynard Smith model, alternately formulated, remains consistent.

Now, applying the same procedure to the Prisoner's Dilemma matrix, we reformulate equation (13a) as:

$$EU(C, I) = EU(D, I) \quad (13a')$$

Explicitly, to find  $x$  such that  $I$  is an ESS, (13a') becomes:

$$xR + (1-x)S = xT + (1-x)P \quad (13b')$$

Again, let  $R = (1/2)(S+T) + (1/2)Z$ , and let  $P = (1/2)(S+T)$ . Then

$$x = (P - S) / [(R + P) - (S + T)] = (P - S) / (R - P)$$

In the numerator,  $P - S = (1/2)(S+T) - S = (1/2)(T - S)$ .

In the denominator,  $R - P = (1/2)Z$ . Thus

$$x = (T - S) / Z \quad (13c')$$

Now the following relation obtains:

$$\begin{array}{l}
 R < T \\
 2R < 2T \\
 2R - (S+T) < 2T - (S+T)
 \end{array}$$

that is,

$$Z < T - S$$

applied to equation (13c'),

$$x > 1 \quad (13d')$$

We cannot, of course, admit a probability greater than unity.

Taken together, equations (13d) and (13d') show that there does not exist a real probability  $x$  ( $0 \leq x \leq 1$ ), such that  $I$  is an ESS in the Prisoner's Dilemma. In other words, there is no evolutionarily stable strategy in the Prisoner's Dilemma, and any assumptions to the contrary must be false. Since neither *DDD* nor *TFT* is an ESS, then neither strategy is capable, or incapable, of 'invading' the other. Thus it transpires that the Axelrod-Hamilton model, which is 'based on the concept of an evolutionarily stable strategy in the context of the Prisoner's Dilemma game' (Axelrod and Hamilton [1981]), is founded on a misconception. The model collapses, and the problems arising from it, as well as the necessity of appealing to sociobiology for a solution, vanish identically.

*Edelstein Centre for the History and Philosophy of Science,  
Technology and Medicine,  
Hebrew University of Jerusalem*

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