The Geometry of Defection: Cascading Mimicry and Contract-Resistant Structures

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Abstract: This paper examines a social contractarian model in which an actor cooperates by mimicry; that is, cooperates just in case there is majority cooperation in his or her vicinity. A computer simulation is developed to study the relation between initial and final proportions of such cooperators, as well as to chart the population dynamics themselves. The model turns out to be non-linear; it embodies a quintessentially chaotic threshold. The simulation also yields other unforeseen results, revealing a "geometry of defection" that unites defecting cells into robust molecular formations which persist within overall cooperative domains, or which under certain conditions undermine cooperativeness entirely. The model thus sheds some light on the structural dimension of mimicry that underlies social communication, conflict and its resolution.

I. Cascading Mimicry

Compliance with social contracts of a generic or implicit kind is often achieved neither by deontological precept nor by teleological expectation; rather, by a kind of cascade effect that proceeds from an ignobly superficial but empirically observable form of mimicry.[1] While it is tempting to attribute high-minded dutifulness, or at least cold-blooded prudence, to those who voluntarily abide by implied contracts,[2] empirical evidence suggests that an individual's compliance, or lack thereof, often depends solely upon a critical threshold of compliance, or lack thereof, in his immediate vicinity.[3]

If one studies the behaviors of social animals that associate in herds, one notes that the phenomenon of stampeding entails precisely this kind of cascade effect. If just a few animals display panic, their behavior can quickly pervade the herd, which may then stampede even if most of its members have not identified, and are not responding to, the initial threat itself. Humans are also capable of panicking and stampeding in just this way, through mimicking the behaviors of conspecifics in their immediate vicinity. Although cascading mimicry is ostensibly a survival mechanism of gregarious animals, a stampede can also kill more conspecifics than the stampede's trigger (e.g. a predator or a lightning bolt) would have. The cascade effect is also patently observable, and of obvious survival value, in schools of fishes and flocks of birds, whose dynamic formations maintain rigid internal metrics along with malleable external envelopes. It appears that a few key individuals initiate changes of direction of motion, and the entire school or flock responds by rapidly cascading mimicry.

Evidence of this cascade effect is also inferable in its absence, through recognition of a threshold below which it never escalates. Consider this empirical social question: how do we describe the behavior of stealing merchandise from stores without paying for it? If relatively few persons engage intermittently in such behavior, we call it "shoplifting"; if relatively many engage in it simultaneously, we call it "looting." At what quantitative threshold does shoplifting cascade into looting? Conduct, if you will, a thought-experiment meant roughly to approximate this threshold. Imagine that you are shopping in a large store, fully intending to pay for the items you select. If you notice one person shoplifting, would you begin to shoplift? Probably not. If you notice one person in ten shoplifting, would you begin to shoplift? Again, probably not. But what if one half of your fellow-shoppers appeared to be shoplifting? Would you not then consider something amiss, and wonder if you should shoplift too? What if nine out of ten persons were shoplifting? At this point, the store is apparently being looted, and you would probably either quit the premises empty-handed or else become a looter yourself. My point is that one does not observe a smooth transition between shoplifting and looting, because the cascade effect is non-linear. Empirically, one witnesses either sporadic shoplifting (normally) or ubiquitous looting (abnormally). Escalation between these states is contingent on a certain threshold being exceeded.

Similarly, in traversing a residential neighborhood one rarely sees a few broken windows, a few scraps of litter, or a few insignias of graffiti: neighborhoods tend either to be either pristine in their neatness, or profane in their decay.[4] Intermediate states of messiness, if not swiftly reduced to neatness, can escalate rapidly into decay. Breakage invites more breakage; litter, more litter; graffiti, more graffiti.

The phenomenon of cascading mimicry has been studied by economists,[5] sociologists,[6] criminologists,[7] psychologists[8] and physicians,[9] in particular manifestations that include vandalism, littering, graffiti-spraying, teenage pregnancy, epidemiology, homicide, and suicide. The phenomenon apparently also applies to fashions, fads, and self-fulfilling prophecies.[10] In a given manifestation, the particular threshold beyond which intermittent occurrence rapidly escalates to epidemic frequency is called by Malcolm Gladwell "the tipping point." Gladwell first encountered the phenomenon as a child, when he attempted to pour ketchup from a bottle, and incorrectly

but understandably assumed a linear causal relation between his tapping on the inverted bottle and the flowing of its contents. His father summarized the actual outcome—and the non-linear cascade effect it represented—with a ditty: "Tomato ketchup in a bottle—None will come and then the lot'll."[11]

Characteristic of the cascade effect in social contexts is its external compulsion on the individual agent, and the asymmetry of decision theory along the spectrum of relative frequency. At the sporadic end, people ostensibly deliberate and choose, according to internal predispositions of the ethical or psychological variety, whether to shop or shoplift. But past the tipping point and toward the epidemic end, there is no sophisticated internal decision principle governing a choice between shopping or looting. People loot because sufficient numbers around them are looting—evidently, people who might not shoplift in normal circumstances might loot in abnormal ones.

This phenomenon is compassed, if not anticipated, by standard Hobbesian contractarianism, in which we emerge from a state of nature by, among other things, renouncing our "natural right" to all things. Recall that Hobbesian natural right is:

> the liberty each man hath, to use his own power, as he will himself, for the preservation of his own nature; that is to say, of his own life; and consequently, of doing any thing, which in his own judgment, and reason, he shall conceive to be the aptest means thereunto.[12]

This would apply to shoplifting which, if everyone applied it, would become looting. We renounce our natural right to steal or loot because, according to Hobbes, it is more commodious, and therefore advantageous, for us to do so.[13] The egoist's long-term expected utilities are maximized by cooperation, not defection, provided that one vital condition is met. Hobbes's second law of nature (meant to effect egress from his state of nature) specifies that proviso:

> that a man be willing, when others are so too, as far-forth, as for peace, and defence of himself he shall think it necessary, to lay down this right to all things; and be contented with so much liberty against other men, as he would allow other men against himself.[14]

This Hobbesian conception, in other words, describes social contractual mimicry: an actor refrains from an action, thereby renouncing an immediate gain in favor of a longer-term expectation, neither because he maintains a high-minded principle proscribing such an action, nor because he has calculated the expected utility of self-restraint, but mainly because he observes or infers that others are refraining from it too. While philosophical scholars have recognized that Hobbes's second law entails a formulation of the N-person Prisoner's Dilemma,[15] and moreover have studied the "free-riding" phenomenon that obtains when a few defectors take advantage of near-ubiquitous cooperation,[16] cascading mimicry itself remains relatively underexamined mathematically and computationally. To shed more computational light on this effect, I have developed a simple model that simulates it.

II. A Computer Model

On a rectangular two-dimensional grid, each cell has exactly eight contiguous neighbors: one above, one below, one left, one right, and one on each corner. These eight neighbors comprise the "vicinity" of a given cell. Cells along the edges are by definition deemed contiguous with cells along opposite edges; thus the grid is a two-dimensional representation of a torus, sliced both longitudinally and transversely then pasted flat. Hence each and every cell has exactly eight neighbors in its vicinity. The extent of our model is a square of 100 x 100 cells, producing a population of 10,000 cells.

The model has two variables: the initial proportion of cooperators in the overall population, and the threshold proportion of cooperative neighbors in a given cell's vicinity sufficient to "elicit" that cell's cooperation via mimicry. I have elected to study primarily the effects of varying initial proportions of cooperators, while holding vicinity thresholds constant. This paper discusses chiefly the results of that study. Secondarily, I conducted simulations showing the evolution of proportionality as a function of varying thresholds; that result is mentioned later in the paper.

Trials are conducted by running a program written in GW-BASIC, whose annotated source code is provided in Appendix One. For each trial, the program requires three inputs: the initial proportion of cooperators in the overall population, the threshold fraction of cooperators in a cell's vicinity sufficient to elicit its cooperation, and the number of generations to be computed before the program halts (i.e. the length of the trial).

In the initial generation, the grid is seeded with the selected initial proportion (p) of cooperators. This is accomplished by sampling each cell in turn, and designating it a cooperator with probability p (or a defector with probability $1-p$). The grid is displayed as a square matrix of cells: cooperators are green; defectors, red. (The matrices appear rectangular on the computer monitor and in the "snapshot" reproductions provided herein owing to standard pixellation differentials in horizontal versus vertical display.) Once the initial generation is in place, subsequent generations are spawned as follows. A cell is sampled randomly, and the fraction of cooperators in its vicinity is computed. If this fraction equals or exceeds the pre-selected threshold, a cooperative state is assigned to that cell; otherwise, a defective state is assigned. Such assignments are naturally independent of the current state of

the sampled cell; they are dependent only upon the overall composition of the cell's vicinity. A generation is defined as 10,000 such random samplings over the whole grid.

Here I interject a note intended to allay any concerns raised by aficionados of programming or other sharp methodologists who may wonder why a generation consists of 10,000 random samplings, as opposed to a deterministic sampling of each of the 10,000 cells. For the record, I essayed both methods, and the outcomes were indiscernible if not identical. Given that equivalent data obtains from either method, random sampling is computationally far less taxing than deterministic sampling. In the deterministic case, the computer needs to retain in memory a complete map of the n^{th} generation while computing and building the map of the $(n+1)^{st}$ generation; then it has to replace the map of the n^{th} with that of the $(n+1)^{st}$ in order to compute the $(n+1)^{nd}$, while at the same time reading and writing appropriate data to update the visual display. This is highly inefficient, especially over a large number of generations. It also interrupts the continuity of the visual display for the experimenter because with each new generation the previous matrix is erased from the screen and the next matrix discretely generated, row-by-row. In the random case, the computer needs to retain and display only one map, which it continuously modifies (as necessary) one cell at a time. This is highly efficient for the computer, and produces an additional benefit for the experimenter: the visual display itself is continuous, and scintillates pleasingly in real time. Thus the cascade effects and overall population dynamics are continuously observable as the generations elapse.

The experimental results reported herein are predicated upon an assumption of democratic minimalism; namely, that with respect to mimicry, people will generally behave as the majority in their vicinity behaves. Thus, for a selected cell to cooperate, the threshold fraction of cooperators in its vicinity must exceed one-half. Since each cell has exactly eight neighbors, at least five of them must be in a cooperative state to elicit cooperation from the selected cell. A heuristic justification for this threshold is plain: in the kind of social situations represented by this model, people will generally mimic the majority, because doing so affords a fortuitous convergence of instinctual gregariousness (or herd mentality, or social contractarianism) and Hobbesian prudence (or psychological egoism, or enlightened self-interest). If a clear majority of shoppers purchases goods, the individual in their midst is likely to purchase too. If a clear majority loots the shop, the same individual in their midst is likely to loot as well. However, if the herd splits into two equal fractions, one of which shops while the other shoplifts, the individual in their midst has no clear majority behavior to mimic. Nor is he being asked to cast a "deciding vote" that would in any way compel more uniform behavior in the aggregate. His condition resembles Buridan's ass among a herd, half of which heads toward one bale, half toward the other. I interpret this state of affairs as the cutting point of Hobbes's second law, at which majority mimicry is rendered impossible *de facto*, and in which case individual prudence trumps social contractarianism *de jure*.

There is also a formal decision-theoretic justification for this location of the cutting point. The generic Prisoner's Dilemma is a conflict between two principles of choice, each leading to a different outcome. Since defection strongly dominates cooperation, the dominance principle prescribes defection, which is individually rational but which, if chosen by both prisoners, leads to a mutually undesirable Nash equilibrium. The principle of maximizing expected utilities prescribes cooperation, which is collectively rational and which, if chosen by both prisoners, results in a mutually desirable Pareto-optimal outcome. However, the expected utility calculus prescribes cooperation if and only if the subjective (or *a priori*) probability that the other prisoner cooperates or defects, conditional on one's cooperation or defection respectively, is greater than one-half. If that probability is less than one half, the calculus prescribes defection, the two decision principles converge, and the dilemma vanishes. If that probability equals exactly one-half, then the expected utilities of cooperating and defecting are equal, and collective rationality's prescription is mute. One then reverts or defaults to individual rationality (i.e. the dominance principle) and defection. For models such as Newcomb's problem[17] and the Tragedy of the Commons,[18] although subjective probability is transposed to empirical frequency, there is no change in the prescriptive cutting point for cooperation. Our model, in turn, imports empirical frequency into a kind of autological Prisoner's Dilemma, in which the payoff for doing what most others do *is* doing what most others do. This, in brief, justifies fixing our empirical cutting point at one-half. A formal demonstration of this argument is given in Appendix Two.

III. Characteristic Trials

Fixing the threshold cooperative fraction as described above, and varying the initial proportion of cooperators in the overall population, one observes, to begin with, that most empirical trials unfold in one of two divergent directions.

First, consider a trial whose initial proportion of cooperators is 60%. As figure 1 shows, after one generation, this proportion has increased slightly to 61%. After 10 generations, however, it has fallen to 38% (figure 2); after 25 generations, to 17% (figure 3); and after 50 generations, to 3% (figure 4). A few more generations will suffice to eliminate the remaining cooperators, and leave the population in a steady state of unanimous defection. The first lesson learned from this trial is that an initial majority of cooperators—in this case 60%—does not guarantee that any cooperation will prevail.

Second, consider a trial whose initial proportion of cooperators is 75%. After one generation, it is still 75% (figure 5). However, by 18 generations it has risen to 96% (figure 6). While this population has also attained a steady state, it contains a 4% minority of permanent defectors. These defectors are clustered in geometric conformations that remain impervious to change. Each of the red cells in these clusters has at least four red neighbors, and therefore remains red (i.e. defective) when sampled in subsequent generations. Similarly, each of the green cells surrounding these clusters has at least five green neighbors, and therefore remains green (i.e. cooperative) when sampled in subsequent generations. The net (geo)graphical effect is the persistence of small islands of defectors in a large sea of cooperators. These cooperators are social contractarians and even though they constitute an overwhelming majority, the defectors have managed to form "contract-resistant" structures.

IV. Typology and Taxonomy of Defecting Structures

A closer examination of these contract-resistant structures, notwithstanding their varying masses and orientations, reveals a definitive and generalizable property of their stable geometries. To explicate this property, we first define the "envelope" of these structures, as a function of their perimeters, in the following way. Where the perimeter of a defecting structure is horizontal or vertical (i.e. where the perimeter cells are aligned in rows or columns), the envelope of that part of the structure is the horizontal or vertical boundary between defecting and cooperating cells. Where the perimeter of a defecting structure is step-wise diagonal (i.e. where the perimeter cells form a facet of the structure neither horizontally nor vertically aligned in the plane), the envelope of that part of the perimeter is the shortest straight line that can be drawn between the adjacent horizontal and vertical facets, which does not transect any defecting cells.

This is much simpler to depict than to describe. Figures 7 and 8 show the envelopes of two stable defecting structures. The envelope of a stable defecting structure is evidently a polygon with the characteristic feature that all its internal angles measure exactly 135 degrees. It must also possess a sufficient mass in order to survive. For example, the envelope of a 5-cell structure with a defecting cell in the center and defecting neighbors north, south, east and west satisfies the angular condition, but this structure's cells cannot resist change to cooperative states. Since a perimeter defector in this structure has only three defecting neighbors, it will cooperate as soon as it is sampled. Thus any such sampling will immediately reduce the center cell's four defecting neighbors to a lesser number, and they too will cooperate when sampled. One finds empirically that the smallest stable defecting mass is the 12-cell structure depicted in figure 6. (We leave the proof that this is a critical mass to a mathematician.) This stable type of defecting structure has an infinitely large taxonomy, restricted in practice only by the size of the grid. Many different stable conformations, exhibiting various kinds of symmetries, can be observed in the figures provided herein, drawn from experimental trials. All possible stable structures should be describable by some inductive formula, or be able to be generated by some algorithm, which might draw upon graph theory, lattice, theory or group theory. We leave this exercise to a mathematician.

If a defecting structure's geometry is such that its envelope contains an internal angle other than 135 degrees, then it is unstable and belongs to one of three unstable types. Each type engenders a distinctly different process. A type 1 unstable defecting structure has either unviable conformation or insufficient mass. Since each of its peripheral cells changes to a cooperative state when sampled, the structure eventually vanishes. Figure 9 depicts some such structures. A type 2 unstable defecting structure has sufficient mass and viable conformation; it sheds peripheral defecting cells until its envelope becomes stable. Figure 10 depicts one such structure. A type 3 unstable defecting structure also has sufficient mass and viable conformation; it accrues defecting cells and thus grows until its envelope becomes stable. Figure 11 depicts one such structure.

V. Collisions and Chaos

Type 3 unstable structures can also give rise to a phenomenon widely observed during experimental computer trials, which I call "collision." As a type 3 structure grows toward stability, it may collide with other defecting structures in the population. Any such collision, even with a stable structure, will result in the formation of a new and larger unstable defecting mass of type 3, which once again will grow toward a stable conformation. Depending on the initial proportion of defectors in the population, and thus on the density of stable defecting structures lying-within the eventual stable envelope of a type 3 growth, collisions can ultimately consume the entire population of cooperators.

This transpires in figures 1-4. Figure 1 shows a proliferation of type 2 and 3 defecting structures in the first generation (39% of the population), which by the tenth generation (figure 2) have collided and coalesced into a sprawling, yet aggregate, type 3 mass (62% of the population), which will shortly and inevitably monopolize the entire grid.

I use the word "inevitably" in a particularly guarded sense because while it is true of the aforementioned experimental run, it is not generally true that all type 3 collisions result in the inevitable extinction of cooperators. Given the methodological programming equivalencies hitherto assumed, it is clear that each experimental trial consists of an initial probabilistic phase, wherein the grid is randomly seeded with the selected proportions of cooperators and defectors, and a protracted deterministic phase, wherein the population is driven to some kind of stasis by the algorithm of mimicry.

Three kinds of static outcomes (i.e., population states that cannot change in subsequent generations) are possible in this model. First, one can arrive at a static population composed purely of defectors (as in the aforementioned trial). Second, one can arrive at a static population composed purely of cooperators. Third, one can arrive at a static population composed of stable defecting structures configured on a stable ground of cooperators-"islands" of defection in a "sea" of cooperation (as depicted in figure 6). Which of these outcomes obtains depends almost strictly on the selected proportions of defectors and cooperators in the initial generation.

Empirically, the initial threshold that determines an end-state of pure defection is any proportion less than 2/3 cooperators. If this initial proportion is chosen, then it is only a matter of time (i.e. number of generations) before type 3 collisions eliminate all the cooperators. The initial threshold that determines an end-state of pure cooperation is any proportion greater than 88% cooperators. If the initial proportion of cooperators (I_c) is selected within this range ($.667 < I_c < .88$), the end-state will be mixed. At the upper threshold, when $I_c = .88$, the end-state will be either purely cooperative, or will contain at most one 12-cell defecting structure, and will require no more than 15 generations to attain stasis. In the latter case, the defectors' proportionality will have dropped by two orders of magnitude, from .12 to .0012. At the lower threshold, when $I_c = .667$, the population dynamics are variable and unpredictable.

This latter process is illustrated in figures 12-14. In figure 12, after 10 generations of a trial in which the initial proportion of cooperators was 67%, the current proportion has risen to 70%. Note, however, the two (unstable) type 3 masses of defectors in the lower right quadrant that are going to collide. In figure 13, by generation 20, these masses have collided, and have formed an aggregate type 3 mass that will collide in turn with other stable defecting structures. The proportion of cooperators has dropped to 66%. In figure 14, the population has attained stasis by generation 175. The proportion of cooperators is 53%; defectors, 47%. The large stable defecting structure that has coalesced out of several type 3 collisions came very close to colliding with two additional smaller structures, which would have precipitated an eventual stable state of pure defection. That was averted merely by the fortuitous initial positions of a few defecting cells, in which a slight variation in the probabilistic initial state could lead to a significant difference in the deterministic end state. Since this is precisely the definition of chaos,[19] the model is unquestionably chaotic at the initial threshold proportion of 66% cooperators. In Gladwell's terminology, this is the model's tipping point.

Figure 15 shows the dynamic contour of this entire family of simulations, which fixes the threshold of cooperative mimicry at 5/8 while varying initial proportions of cooperators. One can see the differential rates at which initial proportions of cooperators attain eventual stasis, either through extinction by defectors, monopolization of defectors, or co-existence with defectors. Given initial proportions of cooperators less than .667, all populations eventually evolve to a state of pure defection. Given initial proportions of cooperators greater than .667, all populations rapidly evolve to a state of either pure cooperation, or overwhelming majority cooperation.

Figure 16 shows the dynamic contour of simulations conducted at the tipping point, or chaotic threshold, of .667. Here the possibilities range from eventual stable states of pure defection to stable states of mixed cooperation and defection, with proportions of cooperators varying from 0% to 70%.

VI. Sensitivity to Cooperative Threshold

If one wishes to study the other parameter, namely cooperative threshold, one can compare the effects of varying it against a fixed initial proportion of cooperators. In Hobbesian terms, this means increasing the diffidence of the individual actor, and hence requiring that he observe a relatively larger fraction of cooperators in his vicinity in order that his cooperation be elicited by mimicry. In our model, suppose we now require that six, seven, or even eight neighbors cooperate in order to elicit the cooperation of a given cell. What initial proportions of cooperators do we need under these new conditions in order for cooperation to prevail or survive? The model responds drastically to such variations, and the results do not augur particularly well for utopians. If the "diffidence factor" is increased by just one increment, such that a selected cell now requires six cooperative neighbors (i.e. a 75% majority in its vicinity) to cooperate itself, the geometry of defection becomes malignantly cancerous. Empirically, we find that any initial proportion of cooperators less than 97% cannot sustain itself under these conditions in which any small contiguous mass of five or more defecting cells grows rapidly and annihilates all cooperators on the grid.

VII. Conclusions

The model reveals that a stable geometry appears to underlie the sociology of defective (i.e., anti-contractarian) behaviors. Although biological, psychological, sociological, criminological, political, and economic explanations can be brought to bear in accounting for behaviors such as shoplifting, the behavior itself persists, notwithstanding such explanations and despite preventive or corrective measures. The geometric properties of the defective structures modeled herein, as well as the thresholds required to sustain them, may inform our understanding of the persistence of certain social behaviors, and thus contribute to the prevention of their escalation via cascading mimicry.

It is also interesting that awareness of this model may itself become a determining factor in social choice. A friend and colleague for whom I ran the computer simulation was, initially unknown to me, deliberating whether to send her daughter to a good public school or a good private one. The quality of education in the private school was known to be somewhat better, but the financial costs were known to be very much higher. The mother was unsure whether drastically higher costs justified somewhat better education, that is, until she witnessed my computer simulation. As she observed the formation and growth of stable defecting structures where initial proportions of cooperators were too low, she suddenly blurted out "That's it! I'm sending my daughter to private school." She concluded that the anti-contractarian behaviors she worried her daughter might mimic would be less prevalent in a more monied population. Indeed, if teenage social behavior can be characterized as an inevitable series of episodes of cascading mimicry, then it makes sense to attempt to influence the behavior itself, or at least to limit the range and frequency of undesirable behaviors in the teenage peer population. This in turn might be accomplished by deselecting one population and selecting another—provided that one has the means. Where one cannot change one's population wholesale, one reverts to the William Bratton expedient of attempting to increase cooperativeness in the given population via conspicuous policing[20] so that the threshold for cascading mimicry is not approached.

Socially, however, the asymmetry of anti-contractarian behavior persists: while one can usually prevent looting, one can apparently never eradicate shoplifting. The geometry of this asymmetry is amply illustrated by the model, whose contract-resistant structures may prove immune to social engineering and impervious to civil persuasion.

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Appendix One: Annotated Source Code (GW-BASIC) for Computer Simulations

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10 INPUT "PROBABILITY, Q-FACTOR, NUMBER";P,Q,NUM *inputs initial cooperative proportion, index that determines cooperative threshold for each cell, number of generations
20 KEY OFF: SCREEN 8: CLS *initializes display
40 C=2: D=4: E= (1+Q)/2 *defines cell colors, and computes cooperative threshold
100 DIM A(101,101) *defines population array
105 RANDOMIZE TIMER
110 FOR J=1 TO 100: FOR K=1 TO 100 *this "for" loop computes initial population
120 IF RND < P THEN A(J,K)=1: N=N+1
130 NEXT K: NEXT J
140 GOSUB 500 *completes torus; i.e. wraps edges of the grid
150 GOSUB 700 *displays initial grid
160 FOR Z=1 TO NUM *this "for" loop computes subsequent generations and displays data
165 LOCATE 5,78: PRINT "GEN": LOCATE 6,77: PRINT USING "####";Z
170 LOCATE 8,79: PRINT "%C": LOCATE 9,78: PRINT USING "###";N/100
175 LOCATE 11,79: PRINT "%D": LOCATE 12,78: PRINT USING "###";100-(N/100)
180 N=0
200 FOR Y=1 TO 10000 *this "for" loop picks cells to be sampled
210 J= INT(RND*100)+1: K=INT(RND*100)+1
220 GOSUB 600 *determines whether a sampled cell cooperates or defects
230 NEXT Y: NEXT Z
260 END *halts program when trial is completed
500 A(0,0)=A(100,100): A(0,101)=A(100,1): A(101,0)=A(1,100): A(101,101)=A(1,1)
510 FOR J=1 TO 100
520 A(0J)=A(100J): A(101J)=A(1J): A(J,0)=A(J,100): A(J,101)=A(J,1)
530 NEXT
540 RETURN
600 F=A(J-1,K-1)+A(J-1,K)+A(J-1,K+1)+A(J,K-1)+A(J,K+1)+A(J+1,K-1)+A(J+1,K)+A(J+1,K+1)
630 IF F/8 > E THEN CIRCLE (J*6,K*2),1,C: PAINT (J*6,K*2),C: A(J,K)=1: N=N+1: ELSE CIRCLE (J*6,K*2),1,D: PAINT (J*6,K*2),D: A(J,K)=0
640 IF J=1 OR K=1 OR J=100 OR K=100 THEN GOSUB 500
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680 RETURN
700 FOR J=1 TO 100: FOR K=1 TO 100
710 IF A(J,K)=0 THEN COL=D ELSE COL=C
720 CIRCLE (J*6,K*2),1,COL
730 PAINT (J*6,K*2),COL
740 NEXT K: NEXT J
750 RETURN
800 FOR J=1 TO 100: FOR K=1 TO 100
810 INPUT#1, A: A(J,K)=A
820 NEXT: NEXT
830 RETURN
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Appendix Two: Conditional Probabilistic Threshold of Cooperation in the PD

The Prisoner's Dilemma payoff matrix is standardly represented as follows:

	P_2 cooperates	P_2 defects
P_1 cooperates	R,R	S,T
P_1 defects	T,S	P,P

where T > R > P > S, in utiles.

T means "temptation to defect"; R, "reward for mutual cooperation"; P, "punishment for mutual defection"; S, "sucker's payoff."

Adopting the usual notation for conditional probabilization, p(c/C) means "the probability that P_1 (prisoner 1) cooperates conditional on P_2's (prisoner 2's) cooperation," and so forth. Then one writes P_2's expected utilities as follows.

Expected utility of cooperation: EUC = p(c/C)R + p(d/C)S.

Expected utility of defection: EUD = p(c/D)T + (d/D)P.

Assuming complete probabilistic dependence, p(c/C) = p(d/D) = 1 and p(d/C) = p(c/D) = 0. Hence EUC = R and EUD = P. Since R > P, the calculus prescribes cooperation.

Assuming partial probabilistic dependence, let p(c/C) = p(d/D) = x and p(d/C) = p(c/D) = 1-x. Then P_2's expected utilities are as follows.

EUC = xR + (1-x)S

EUD = (1-x)T +xP

For cooperation, we require that EUC > EUD, or

(R-P)/(T-S) > (1/x)-1

Since T > R and P > S, the left side is necessarily less than unity. Therefore, the inequality cannot possibly hold unless x is greater than 1/2 (Q.E.D.).

Note that a celebrated case obtains if we constrain S=0 and T=R+P. These are the payoffs for Newcomb's problem, and the same result obtains: the calculus prescribes that the player choose box two only, just in case the demon's observed relative frequency of correct predictions is greater than 1/2. Note also that David Lewis got this taxonomy wrong: while a Newcomb problem is a Prisoner's Dilemma, the converse does not hold; a Prisoner's Dilemma is not necessarily a Newcomb problem.[21]

In fine, the calculus prescribes that P_1 cooperate just in case the conditional probability that P_2 cooperates is greater than 1/2. Newcomb's problem transposes an assumed conditional probability into an observed empirical frequency, without affecting the calculus itself. Making a similar transposition to our N-player model, a given agent cooperates just in case the observed proportion of cooperators in his immediate vicinity is greater than 1/2. This details the formal justification for the said assumption in our model.

Notes

1. Nancy Glance and Bernardo Huberman, "The Outbreak of Cooperation," *Journal of Mathematical Sociology* 17 (1993): 281–302; and idem., "Dynamics of Social Dilemmas," *Scientific American*, March 1994, 76–81.

2. Lou Marinoff, "The Tragedy of the Coffeehouse: Costly Riding, and How to Avert It," *Journal of Conflict Resolution* 43 (1999): 434–450.

3. Malcolm Gladwell, "The Tipping Point," *The New Yorker*, 3 June 1996, 32–38; and idem, *The Tipping Point* (New York: Little, Brown & Company, 2000).

4. This has been a major concern in New York City; see George Kelling and Catherine Coles, *Fixing Broken Windows* (New York: Touchstone, 1996).

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